A System for Rapid Measurements of RF and Microwave Properties Up to 1400°C

Part 1: Theoretical Development of the Cavity Frequency-Shift Data Analysis Equations

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A system has been developed, based on the cavity frequency shift technique, for doing rapid measurements of the scalar permittivity and permeability of samples. The basic cavity frequency shift equation is reviewed, and the practical formulae, which are used for determining the values of ε and μ from the cavity frequency and Q shift, are developed.

Key Words:
Cavity perturbation, Permittivity measurement, Permeability measurement.

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A simple system (Figure 1) has been developed for doing rapid studies of complex scalar permittivity and permeability between 50 MHz and 2450 MHz, and up to a temperature of 1400°C. The traditional resonant cavity frequency shift technique is used, where a sample is introduced into either the electric or magnetic field region of a cavity and the change in frequency and Q are related to the permittivity (ε' and ε") or the permeability (μ' and μ") of the sample. A small cylindrical sample, mounted in a thin-walled low loss sample holder tube, is heated and then inserted into a well-cooled resonant cavity whose resonant frequency and Q are determined. The data analysis assumes the general formalism of resonant cavity perturbation theory, but relies, for absolute calibration, either on measurements of known materials or on exact calculations of the frequency shift and Q using the 2-D code SEAFISH [de Jong et al., 1992].

The "cavity perturbation" technique has the reputation of limited accuracy, possibly because of the use of the term "perturbation" in common reference to it. In fact, the basic equation for the frequency shift of a resonator caused by putting a sample in it is exact, and known approximations are generally used when applying the equation to practical analysis. We have reviewed the fundamental equations and, for completeness, show the development of approximate formulae which we use for data analysis. Another paper of this group [Adams et al., 1992] looks at the detailed comparison of the present derived data analysis formulae with exact numerical calculations.

Theory for the Determination of Complex Electric and Complex Magnetic Susceptibilities Using the Resonant Cavity Frequency Shift Technique

The resonant cavity frequency shift formula has been derived and presented by many authors, including Bethe and Schwinger [1943], Maier and Slater [1952], Von Aulock and Rowen [1957], Spencer et al. [1965], and Altschuler [1963]. The generalized formulation shows that, in principle, both ε' and ε" can be determined for a sample by the measurement of the frequency shift and change in Q produced by introducing the sample into a resonant cavity. In practice, the theory contains a dependence on the shape of the sample which can

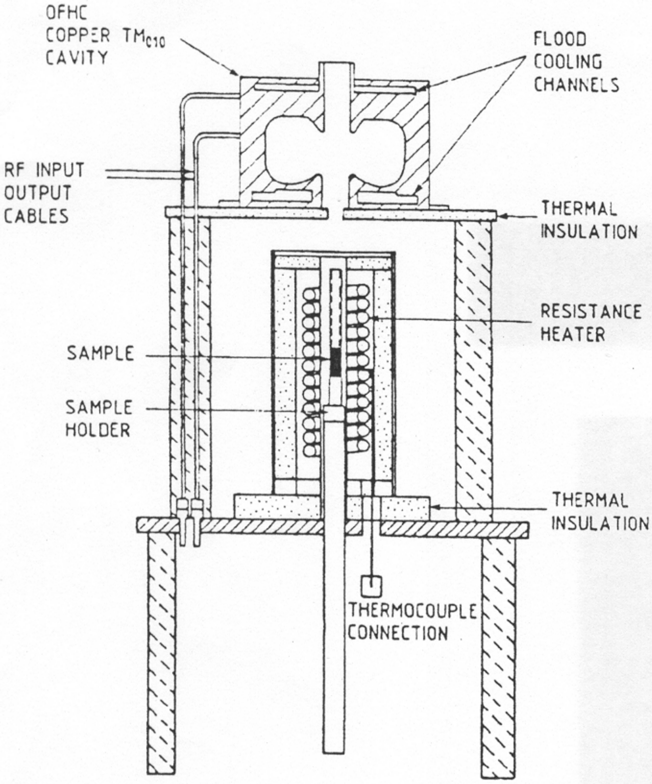


FIGURE 1: Schematic drawing of a simple apparatus for measuring the high temperature complex dielectric constant of inserting a hot sample rapidly into a well-cooled cylindrical cavity driven in the TM_{010} mode.

be analytically solved only for ellipsoids of rotation [Maier and Slater, 1952]. Various limiting shapes (long needles, thin discs) are useful for diagnostics work but are of limited use for quantitative work [Maier and Slater, 1952]. The spherical shape, for which analytic solutions exist, is not convenient for manufacturing test samples.

These problems, coupled with a lack of appreciation of the complete theory, have led to a general mistrust of, or at least uneasiness with, what is often called the "cavity perturbation technique". The following review is intended to provide an understanding of the exact analytical theory and of the approximate form of the relations often used in data analysis.

Resonant Cavity Frequency Shift Formulae

The basic equations for the change in "complex" frequency caused by replacing sample #1 with sample #2 in a resonant cavity with lossless metal walls were derived following the article in Sucher and Fox [Altschuler, 1963], and one form is

$$\left(\frac{\omega_2-\omega_1^*}{\omega_1^*}\right) = \frac{-\int_{V_c}\left[\left(\mu_2-\mu_1^*\right)\vec{H}_2\cdot\vec{H}_1^* + \left(\varepsilon_2-\varepsilon_1^*\right)\vec{E}_2\cdot\vec{E}_1^*\right]dv}{\int_{V_c}\left[\varepsilon_1^*\vec{E}_1^*\cdot\vec{E}_2+\mu_1^*\vec{H}_1^*\cdot\vec{H}_2\right]dv} \tag{1}$$

where V_c is the cavity volume; subscripts 1 and 2 denote values with samples 1 and 2, respectively; where ω, ε, μ, E, H are the resonant frequency, permittivity, permeability, electric and magnetic fields; and * denotes the complex conjugate. All these quantities are complex and the vectors have an $e^{j\omega t}$ time dependence. It is important to note that Equation (1) is not an approximation.

The complex frequency, ω, can be explicitly represented by

$$\omega = 2\pi f + j\,\alpha\,, \quad \alpha = 2\pi f/\,(2Q) \tag{2}$$

where α is the usual rate constant for the decay of the fields in the cavity. The left hand side of Equation (1) is then expanded as

$$\frac{\omega_2-\omega_1^*}{\omega_1^*} = \left[\frac{f_2-f_1}{f_1} + j\left(\frac{f_2}{f_1}\bullet\frac{1}{2Q_2}+\frac{1}{2Q_1}\right)\right]\bullet\left[1-\frac{j}{2Q_1}\right]^{-1} \tag{3}$$

where $1/Q_2$ and $1/Q_1$ include only loss contributions from the samples.

The complex permittivity and permeability are usually represented with so-called real and absorptive (imaginary) parts, with

$$\varepsilon = \varepsilon' - j\varepsilon'' = |\,\varepsilon\,|\, e^{-j\delta_E}$$
$$\mu = \mu' - j\mu'' = |\,\mu\,|\, e^{-j\delta_M} \tag{4}$$

where, for example, δ_E is the amount that the phase angle between the total current (including displacement current) in the material and the electric field in the material differs from 90° if δ_M is zero.

Thus

$$\tan\delta_E = \frac{\varepsilon''}{\varepsilon'} \text{ and } \tan\delta_M = \frac{\mu''}{\mu'}\;. \tag{5}$$

The complex scalar susceptibilities are similarly defined, with

$$X_e = \frac{\varepsilon - \varepsilon_0}{\varepsilon_0} = X_e' - j\, X_e'' ;$$

$$X_m = \frac{\mu - \mu_0}{\mu_0} = X_m' - j\, X_m'' . \qquad (6)$$

If sample #1 permeability and dielectric constant are those of free space (i.e., an empty cavity with $\mu_1 = \mu_0, \varepsilon_1 = \varepsilon_0$), then ω_1 is real ($1/2Q_1 = 0$) and, if sample #2 volume (V_s) is small compared to the cavity volume,

$$\left(\frac{\omega_2 - \omega_1}{\omega_1}\right) \approx \frac{-\int_{V_s}\left[X_m \mu_0 \vec{H}_2 \bullet \vec{H}_1^* + X_e \varepsilon_0 \vec{E}_2 \bullet \vec{E}_1^*\right] dv}{\int_{V_c}\left(\varepsilon_0 \vec{E}_1^* \bullet \vec{E}_1^* + \mu_0 \vec{H}_1 \bullet \vec{H}_1^*\right) dv} \qquad (7)$$

where the integral in the denominator is four times the initial stored energy, as $\vec{E}_1$ and $\vec{H}_1$ are peak field values. This is an approximation that neglects the field change over the small volume of the sample relative to the field integral over the large volume of the empty cavity. The maximum fractional error produced by this approximation occurs for large ε and is $\leq 10\, V_s/V_c$. For normal measurement conditions, this is much less than 1%. An iterative analysis technique would reduce errors introduced by this approximation.

When applied to measurements on an actual cavity, the left hand side of Equation (7) must be written as

$$\frac{\omega_2 - \omega_1}{\omega_1} = \frac{\Delta f}{f_1} + j \bullet \left(1 + \frac{\Delta f}{f_1}\right) \bullet \left(\frac{1}{2Q_2}\right) \approx$$

$$\frac{\Delta f}{f_1} + j\left[\frac{1}{2Q_{L,S}} - \frac{1}{2Q_{L,E}}\right] \qquad (8)$$

where $Q_{L,E}$ is the loaded Q of the empty cavity, $Q_{L,S}$ is the loaded Q of the cavity with the sample in it. The approximately is to the order of $\Delta f/f$, or usually less than 0.1%. Equation (8) shows how the contributions to the resonance width resulting from (a) the finite conductivity of the metal walls of the cavity and (b) the coupling to the driving and sensing devices are subtracted, resulting in the pure loss contribution from the sample.

The resonance condition in a single mode cavity necessarily produces spatial separation of magnetic and electric field maxima. For example, in a TM_{010} mode in a cylindrical cavity, the electric field is a maximum on axis, while the magnetic field goes exactly to zero on-axis. Thus a sample on axis in such a mode distribution may in most cases be treated as being in pure electric field only.

Under these conditions, Equation (7) reduces to

$$\frac{\Delta f}{f_1} + j\left(\frac{1}{2Q_2}\right) \approx -X_e \bullet \frac{\int_{V_s} \vec{E}_S(\vec{r}) \bullet \varepsilon_0 \vec{E}_1^*(\vec{r})\, dv}{2\int_{V_c} \varepsilon_0 \vec{E}_1(\vec{r}) \bullet \vec{E}_1^*(\vec{r})\, dv} \qquad (9)$$

For a sample placed in a region where the magnetic field is dominant, an "effective" scalar permeability may be extracted using

$$\frac{\Delta f}{f_1} + j\left(\frac{1}{2Q_2}\right) \approx -X_m \bullet \frac{\int_{V_s} \vec{H}_S(\vec{r}) \bullet \mu_0 \vec{H}_1^*(\vec{r})\, dv}{2\int_{V_c} \mu_0 \vec{H}_1(\vec{r}) \bullet \vec{H}_1^*(\vec{r})\, dv} \qquad (10)$$

where H_1 is the unperturbed cavity RF field. (The magnetization is, in general, a tensor quantity with off-diagonal elements, and formulae have been developed for the full tensor treatment [Von Aulock and Rowan, 1957].)

If a sample whose shape is an ellipsoid of rotation with eccentricity e is inserted into what initially was a region of static uniform electric or magnetic field, then the field inside the sample will be constant and uniform in direction (see Figure 2).

The expression for the static electric field in such a sample with the "a" symmetry axis aligned in the direction of the initial electric field is [Straton, 1941]

$$E_S = \frac{E_1}{1 + X_e Fsh_e} \tag{11}$$

where Fsh_e, the shape factor for static electric fields, is given by

$$Fsh_e = \frac{(1-e^2)}{e^2}\left[\frac{1}{2e}\ln\left(\frac{1+e}{1-e}\right) - 1\right] \text{ for } e > 0 \tag{12}$$

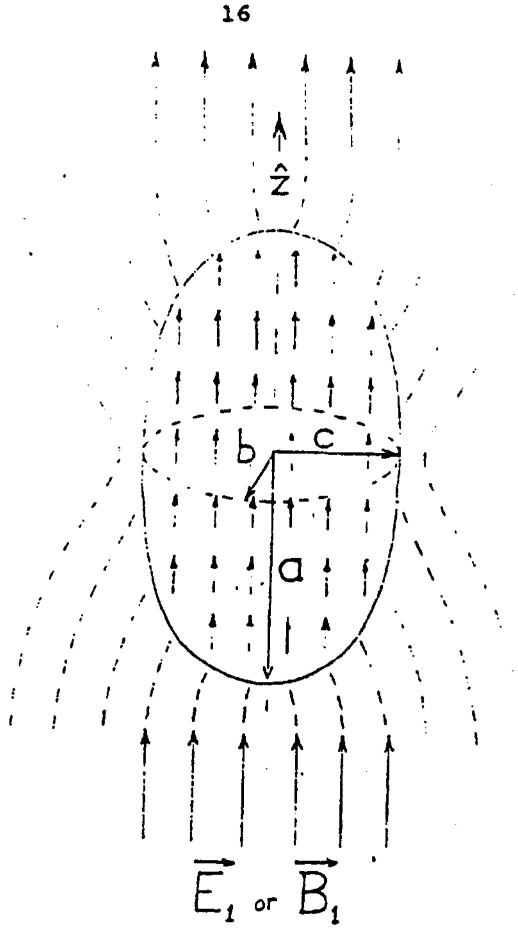
As $e \rightarrow 0$ (i.e., a sphere), $Fsh_e \rightarrow 1/3$.

In the limit when the characteristic sample dimension is small relative to the characteristic wavelength in the sample, the RF field will be approximately the same as in the static case. Thus, inserting the expression for the static internal field into the RF frequency-shift formula and assuming the sample is small relative to the RF wavelength in the sample material and that the empty cavity electric fields are uniform over the small volume which the sample will occupy, one has

$$\frac{\Delta f}{f_1} + j\left(\frac{1}{2Q_2}\right) \approx \frac{-X_e}{(1 + X_e Fsh_e)} \bullet \frac{V_S}{V_C} \bullet \left[\frac{|\vec{E}_1(\vec{r}_S)|^2}{2\int_{V_C} |\vec{E}_1(\vec{r})|^2 \, d\left(\frac{v}{V_C}\right)}\right] \tag{13}$$

where r_s is the location of the sample in the cavity. The quantity in the final square brackets is real, is a function only of the cavity shape and the specific cavity mode, and will be a calibration constant.

Exceptions to this formulation occur when the assumption of a single dominating electric (or magnetic) field is not sufficiently accurate. This is the case for materials for which one of the complex frequency shift components produced by the dominant field is so small that the contribution from the small field influence is still significant. An example of this occurs for dielectric property measurements on very low dielectric loss ferrites which, however, have very large magnetic losses; i.e., $\mu'' >> \varepsilon''$. Especially in the case where ε' is large, the displacement current in the finite radius sample can produce sufficient magnetic field in the sample to incur measurable magnetic loss. Under such conditions, the magnetic field in a rotationally symmetric sample on-axis in a TM_{ono} cavity is given in cylindrical coordinates by



Ellipsoid of Rotation

$a \geq b = c$

$$e = \left(1 - \frac{c^2}{a^2}\right)^{1/2} \qquad V_S = \frac{4}{3}\pi a c^2$$

FIGURE 2: Illustration of the electric fields in an ellipsoid of rotation introduced in an initially uniform field region.

$$\vec{H}_S(r) = \frac{-j\omega\varepsilon_r^s \varepsilon_o r\,(\vec{E}_S(r=0) \times \hat{r})}{2} \tag{14}$$

where ε_r^s is the relative complex dielectric constant of the sample. Substituting into the numerator of Equation (7) and making the previously described approximations and substitutions, one obtains a modified version of Equation (13).

$$\frac{\Delta f}{f_1} + j\left(\frac{1}{2Q_2}\right) \approx \frac{-\left[X_e + \left[\frac{\varepsilon_s}{\varepsilon_o}\bullet\frac{\pi^2 r_s^2}{2\lambda_f^2}\right]\bullet X_m\right]}{\left(1 + X_e\, Fsh_e\right)}$$

$$\bullet\frac{V_s}{V_c}\bullet\left[\frac{\left|\vec{E}_1(r=0)\right|^2}{2\int_{V_c}\left|\vec{E}_1(r)\right|^2 d\left(\frac{v}{V_c}\right)}\right] \qquad (15)$$

where ε_s is the complex dielectric constant of the material
r_s is the radius of the cylindrical sample
λ_f is the free space wavelength of TEM radiation at the measurement frequency.

The commonly used, simple form for the dielectric shift, Equation (13), can be rewritten to more easily demonstrate its properties:

$$\frac{\Delta f}{f_1} + j\left(\frac{1}{2Q_2}\right) \approx \frac{-X_e}{\left(1 + Fsh_e\, X_e\right)}\bullet\frac{V_s}{V_c}\bullet[A] \qquad (16)$$

where the quantity in square brackets is a real constant which depends only on the shape of the empty cavity fields.

It should be emphasized that both the real and absorptive parts of the dielectric constant are given by Equation (16) using the single value of the real constant, A. If, as is often done, the real and imaginary parts are separated into two equations, there is still only one common calibration constant. Thus the calibration constant can be determined experimentally using low loss materials and then later applied exactly to lossy materials! For large X_e the fractional frequency shift reaches a limiting value which depends on the sample volume and shape; a metallic sample of the same size and shape produces this maximum frequency shift. For example, a long thin ellipsoid ($Fsh_e < 0.1$) produces a larger shift than a sphere of the same volume.

The magnetic field in a sample is given by an expression similar to Equation (11) when the sample is an ellipse of rotation with the "a" symmetry axis aligned in the direction of the initial magnetic induction.

$$H_s = \frac{H_1}{1 + X_m Fsh_m} \qquad (17)$$

where, because of the different boundary conditions, the magnetic shape factor is different from the electric shape factor.

$$Fsh_m = \frac{(1-e)^2}{2e^2}\left[\frac{1}{1-e^2} - \frac{1}{2e}\ln\left(\frac{1+e}{1-e}\right)\right] \quad \text{for } e > 0 \qquad (18)$$

For $e \rightarrow 0$ (i.e., a sphere), $Fsh_m \rightarrow 1/3$.

Inserting the expression for the internal magnetic field into the frequency shift formula, one has

$$\frac{\Delta f}{f_1} + j\left(\frac{1}{2Q_2}\right) \approx \frac{-X_m}{\left(1 + Fsh_m\, X_m\right)}\bullet\frac{V_s}{V_c}$$

$$\bullet\left[\frac{\left|\vec{H}_1(\vec{r}_s)\right|^2}{2\int_{V_c}\left|\vec{H}_1(\vec{r})\right|^2 d\left(\frac{v}{V_c}\right)}\right] \qquad (19)$$

As with the electric case, this may be rewritten to more easily demonstrate its properties:

$$\frac{\Delta f}{f_1} + j\left(\frac{1}{2Q_2}\right) \approx \frac{-X_m}{\left(1 + Fsh_m\, X_m\right)}\bullet\frac{V_s}{V_c}\bullet[B] \qquad (20)$$

The quantity in square brackets is a real constant, as is the shape factor. Again, the maximum frequency shift for large X_m is dependent only on the sample size and shape, and is larger for a sphere ($Fsh_m = 1/3$) than for a long thin ellipsoid of the same volume ($Fsh_m = 0.45$ for a major to minor axis ratio, c/a = ~~m~~ 1/3.5).

The frequency shift produced by a high-conductivity metallic ellipsoid in a magnetic field is obtained by putting $X_m = -1$. In this case, the RF currents induced on the surface of the metal generate an RF magnetic field distribution which, when summed with the applied RF magnetic field, produces zero magnetic field in the bulk of the metal, equivalent to $\mu = 0$ (that is, $X_m = -1$). Thus the sign of the frequency shift caused by inserting a metal perturber into a magnetic field is opposite to that produced by a permeable material.

The accuracy with which the cavity frequency-shift technique can, in principle, measure complex susceptibilities is clearly a function of the sample dimensions (relative to the cavity dimensions), the sample shape, and the absolute value of the sample susceptibility. For cylindrical samples (rather than ellipsoids of rotation) the shape factor presented here is not applicable. In fact, it must be reduced by almost a factor of two to yield the correct frequency shift for the case of a cylindrical sample with length to diameter ratio, $(\ell/d) = (a/c) = 3.5$. This problem of shape factor "correction" for practical samples is addressed in a further paper [Adams et al., 1992], where, by comparison with exact numerical calculations, if Equation 16 is suitably calibrated for sample shape, the error in the measured permeability is ≤ 5% if ε_r' and ε_r'' are less than 30.

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