WHAT IS THE BEST MICROWAVE ABSORBER?

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ABSTRACT

The absorption of microwave power by samples with different shapes and dielectric properties was studied for single-mode excitation of a microwave applicator. Exact calculations were done of the quality factor, Q and the frequency shift of a cylindrically-symmetric cavity with a small, cylindrically-symmetric sample on-axis in the cavity, demonstrating the influence of sample shape and dielectric properties on the power absorption per unit volume of the sample.

An approximate, but very general, analytical formalism was developed for calculating the power absorbed by the sample and thus the contribution of the sample to the Q of the cavity. This simple formalism is shown, by comparison with the exact calculations, to be quite accurate over a very broad range of properties. Thus, it can be used to calculate the power absorbed by samples which are smaller than the internal wavelength, and also to gain insight into the factors that influence the microwave absorption.

INTRODUCTION

Often, those who are learning about the use of microwaves for processing materials will ask, “What is the best microwave absorber?" The traditional reply is that the rate of microwave absorption depends on the magnitude of the dielectric loss factor and, if the dielectric constant is large, on the shape of the material. Such a response is not very satisfying to the novice, and generally does not produce much understanding or insight. Any attempt to provide a more complete answer for a multimode cavity, especially giving quantitative numbers, is foiled by the complexity and variability of the mode distributions. However, for a single mode electric field distribution, the problem is readily solved. The results are quantitatively applicable to a single-mode industrial applicator, and give some insight into multimode power absorption.

In this paper we demonstrate and discuss results for power absorption in a single-mode cavity resonator, and also outline a simple analytic formalism that allows one to easily calculate quantitative results, at least for small samples. To study the problem, one would ideally insert **many** samples of different shapes and dielectric constants into the peak electric- field region of a single-mode cavity, where the electric field is relatively uniform. As this is not practical, we have done the equivalent “computer experiments” in which the cavity wall losses were set to zero, leaving only the influence of the samples. The code SEAFISH was used, which has been extensively verified and shown to accurately model axially-symmetric, "material-in-cavity" problems[1,2]. It was desirable to have results for the two industrial frequencies, 915 and 2450 MHz, so the code was used to design a cavity (Fig. 1) which is resonant at both frequencies.

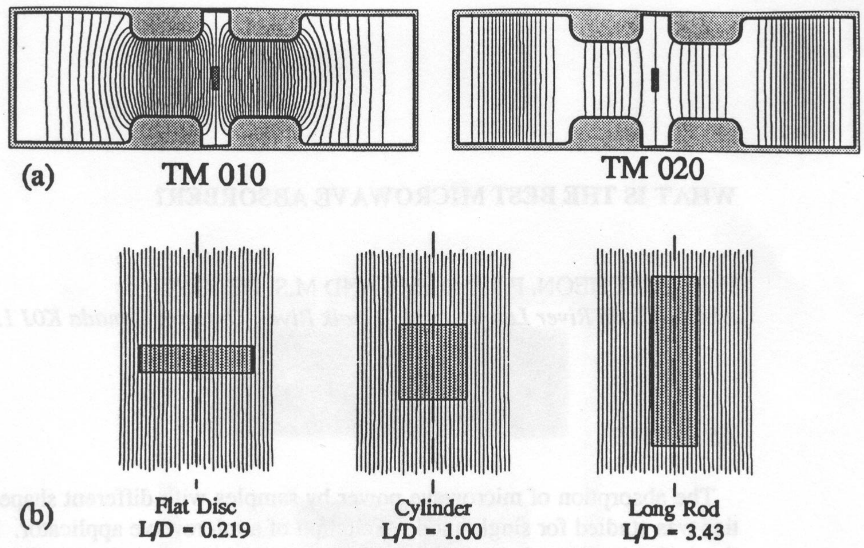


Figure 1. (a) A cylindrical cavity with TM resonances at both 915 and 2450 Mhz. (b) The three sample shapes used for calculations.

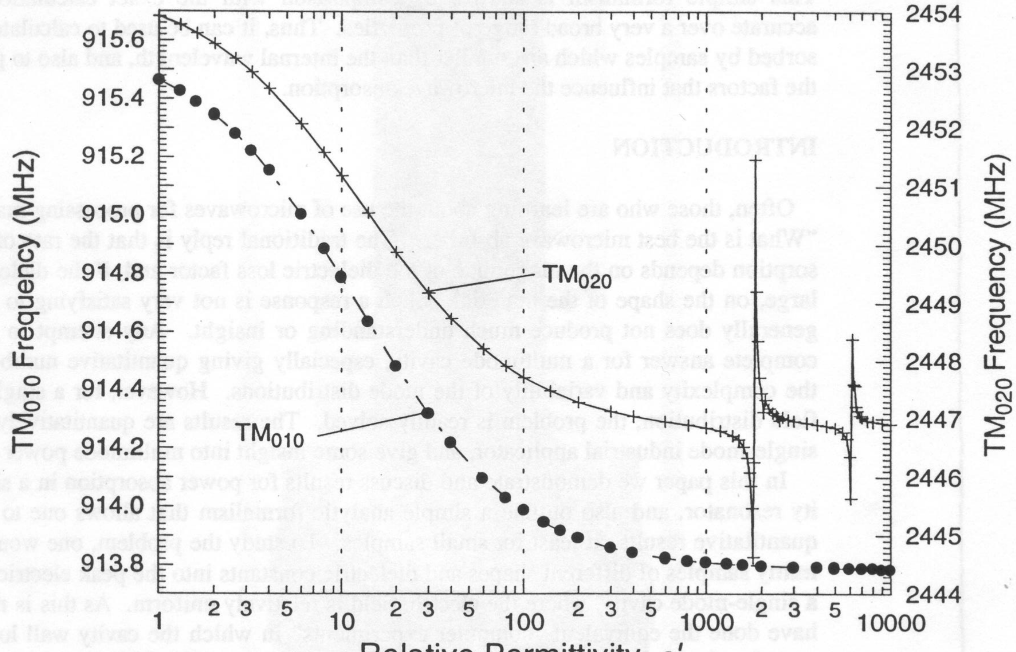


Figure 2. The calculated resonant frequencies of the cavity with a long rod sample of lossless material (ε''_r=0) as a function of ε'_r.

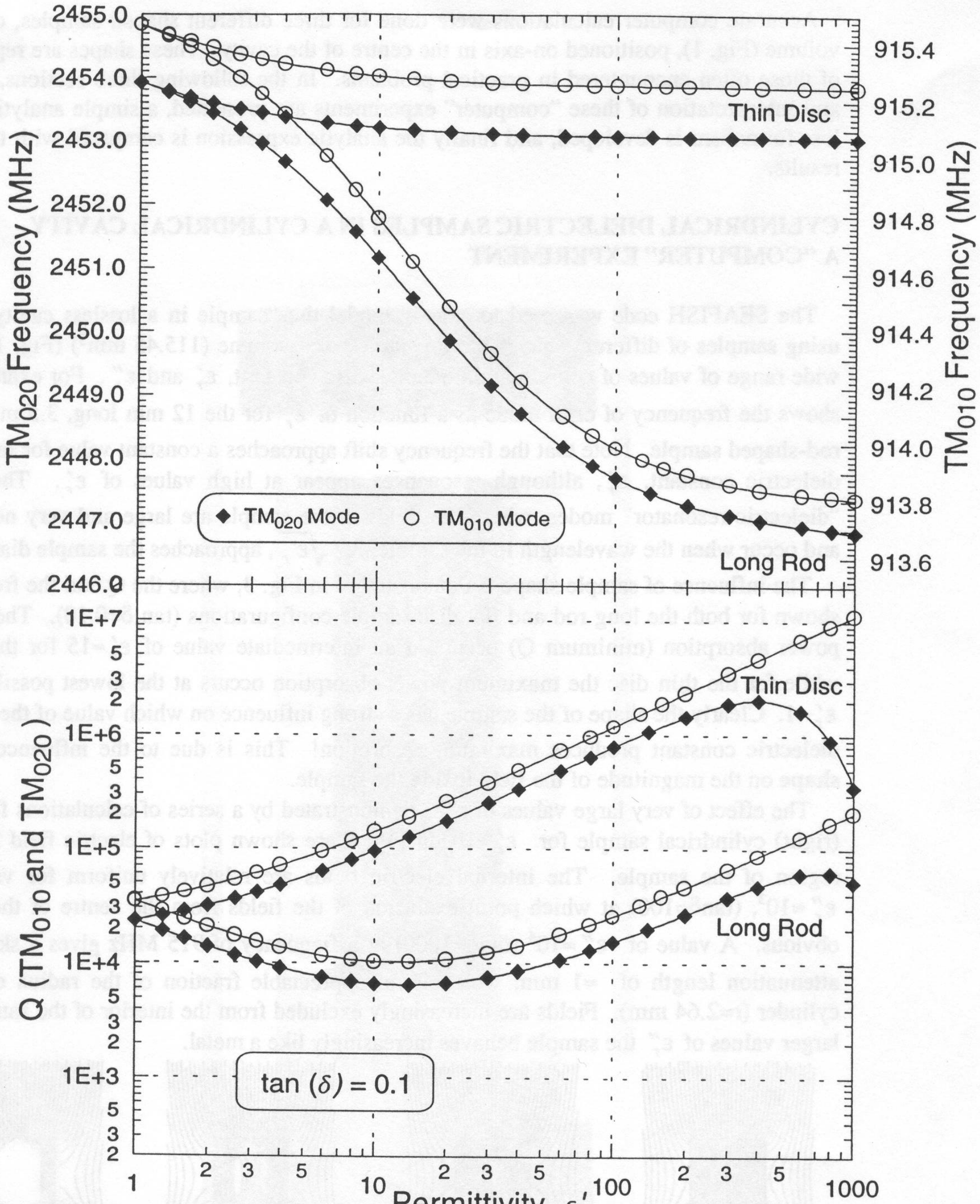


Figure 3. The calculated frequency and Q as a function of ε'_r for the extremes of sample shape.

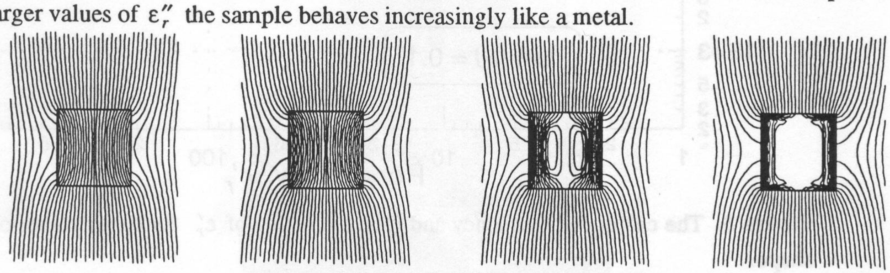
Accurate computer calculations were done for three different shaped samples, of the same volume (Fig. 1), positioned on-axis in the centre of the cavity. These shapes are representative of those often encountered in practical problems. In the following three sections, the results and interpretation of these "computer" experiments are presented, a simple analytic dielectric loss formalism is developed, and finally the analytic expression is compared with the accurate results.

CYLINDRICAL DIELECTRIC SAMPLES IN A CYLINDRICAL CAVITY - A "COMPUTER" EXPERIMENT

The SEAFISH code was used to exactly model the "sample in a lossless cavity" problem, using samples of different shapes but the **same** fixed volume (115.45 mm^3) (Fig. 1) and for a wide range of values of complex relative dielectric constant, ε_r' and ε_r''. For example, Fig. 2 shows the frequency of each mode as a function of ε_r' for the 12 mm long, 3.5 mm diameter rod-shaped sample. Note that the frequency shift approaches a constant value for large relative dielectric constant, ε_r', although resonances appear at high values of ε_r'. These are the "dielectric resonator" modes, where the fields in the sample are large and very non-uniform, and occur when the wavelength **in** the sample, $\lambda / \sqrt{\varepsilon'_r}$, approaches the sample diameter.

The influence of sample shape is demonstrated in Fig. 3, where the Q and the frequency are shown for both the long rod and flat disc sample configurations (tan δ=0.10). The maximum power absorption (minimum Q) occurs at an intermediate value of $\varepsilon_r' \approx 15$ for the long rod, while for the thin disc the maximum power absorption occurs at the lowest possible value of $\varepsilon_r' \approx 1$. Clearly the shape of the sample has a strong influence on which value of the real part of dielectric constant produces maximum absorption! This is due to the influence of sample **shape** on the magnitude of the field **inside** the sample.

The effect of very large values of ε'' is demonstrated by a series of calculations for the short (right) cylindrical sample for $\varepsilon_r' \approx 10$. In Fig. 4 are shown plots of electric field lines in the region of the sample. The internal electric fields are relatively uniform for values up to $\varepsilon_r'' \approx 10^3$, (tan$\delta$=100) at which point exclusion of the fields from the centre of the sample is obvious. A value of $\varepsilon_r'' \approx 10^4$ (tanδ=1000) at a frequency of 915 MHz gives a skin depth or attenuation length of $\approx$1 mm, which is an appreciable fraction of the radius of the right cylinder (r=2.64 mm). Fields are increasingly excluded from the interior of the sample, and at larger values of ε_r'' the sample behaves increasingly like a metal.



tan(δ) = 0.1   tan(δ) = 100   tan(δ) = 1000   tan(δ) = 10000

Figure 4. Calculated (SEAFISH) electric field lines in and around the sample for ε'=10, demonstrating more uniform fields for low loss and field exclusion from the sample core for high loss.

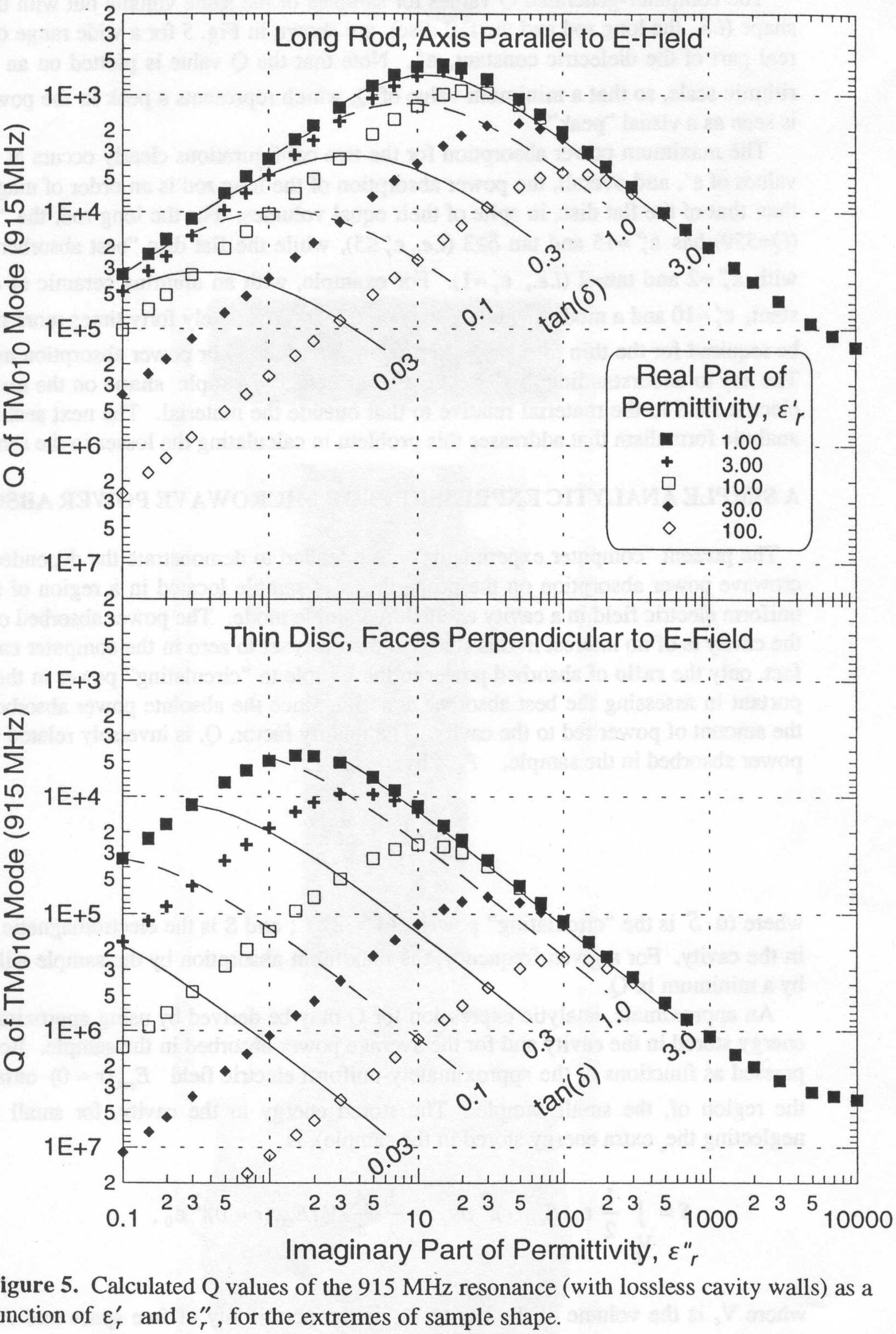


Figure 5. Calculated Q values of the 915 MHz resonance (with lossless cavity walls) as a function of ε_r' and ε_r'', for the extremes of sample shape.

The computer-generated Q values for samples of the same volume but with the extremes of shape (*i.e.*, the long rod and the flat disc), are shown in Fig. 5 for a wide range of values of the real part of the dielectric constant, ε_r'. Note that the Q value is plotted on an inverted logarithmic scale, so that a minimum value of Q, which represents a peak in the power absorption, is seen as a visual "peak".

The maximum power absorption for the two configurations clearly occurs at quite different values of ε", and overall, the power absorption of the long rod is an order of magnitude greater than that of the flat disc, in spite of their equal volumes. For the long rod, the "best absorber" (Q=550) has $\varepsilon_r'' \approx 15$ and tan $\delta \geq 3$ (*i.e.*, $\varepsilon_r' \leq 5$), while the flat disc "best absorber" (Q≈4400) is with $\varepsilon_r'' \approx 2$ and tan≈2 (*i.e.*, $\varepsilon_r' \approx 1$). For example, with an alumina ceramic of dielectric constant, $\varepsilon_r' \approx 10$ and a modest value of tan $\delta \approx 0.03$, approximately forty times more material would be required for the thin disc shape to produce the same Q or power absorption as the long rod. The key to understanding this effect is the influence of sample **shape** on the magnitude of the electric field in the material relative to that outside the material. The next section outlines an analytic formalism that addresses this problem in calculating the losses in the sample material.

A SIMPLE ANALYTIC EXPRESSION FOR MICROWAVE POWER ABSORPTION

The present "computer experiments" are intended to demonstrate the dependence of the microwave power absorption on the properties of a sample located in a region of approximately uniform electric field in a cavity excited in a single mode. The power absorbed on the **walls** of the cavity is of no interest in this study, and so was set to zero in the computer calculations. In fact, only the **ratio** of absorbed power in the sample to "circulating" power in the cavity is important in assessing the best absorber material, since the absolute power absorbed depends on the amount of power fed to the cavity. The quality factor, Q, is inversely related to the average power absorbed in the sample, P_{AV}, by

$$\left(\frac{1}{Q}\right) = \frac{P_{AV}}{\omega \cdot S}, \qquad (1)$$

where $\omega \cdot S$ is the "circulating" power, $\omega = 2\pi f$, and S is the electromagnetic energy stored in the cavity. For a given frequency, the maximum absorption by the sample will be indicated by a minimum in Q.

An approximate, analytic expression for Q may be derived by using approximations for the energy stored in the cavity and for the average power absorbed in the sample. Both may be expressed as functions of the approximately-uniform electric field $E_{ext}(r \approx 0)$ **outside** of, but in the region of, the small sample. The stored energy in the cavity, for small samples (*i.e.*, neglecting the extra energy stored in the sample), is

$$S \approx \int_{V_C} \frac{1}{2}\varepsilon_0 |E_{ext}(r)|^2 dv_c \approx \frac{1}{2}\cdot\frac{A}{4}\cdot V_c |E_{ext}(r \approx 0)|^2 \cdot \varepsilon_0 , \qquad (2)$$

where V_c is the volume of the cavity, ε_o is the permitivity of free space and A is the cavity electric-field shape factor, a unitless constant of order unity which depends **only** on the spatial distribution of electric field in the cavity. For a rectangular waveguide cavity, A=1 for the

TE_{10n} modes. For a pure cylindrical cavity, A=1.078 for the TM_{010} mode and A=0.463 for the TM_{020} mode. It should be noted that, throughout this paper, all electric fields are the peak temporal values.

The problem of calculating the energy absorbed by the sample for a given stored energy in the cavity (*i.e.,* for a given electric field **outside** the sample) has two parts: *(1)* determine the field inside the sample for a given **external** field, and *(2)* calculate the absorption in the sample for the determined value of the **internal** field.

In the situation of a traveling wave incident on a sample, part (1) of the calculation is the equivalent of solving for the reflection and transmission at the sample boundary. However, the present, standing-wave situation is the equivalent of equal and opposite traveling waves incident on opposite sides of the sample with specific phase relationships, and so is quite a different problem.

If the sample shape were an ellipsoid of rotation and the fields were uniform in the region of the samples, then the relative electric fields inside and just outside the sample would be given **exactly**[3] by the expression

$$\frac{E_{\text{int}}}{E_{\text{ext}}} = \frac{1}{1 + F_{\text{sh}}\chi} \tag{3}$$

where χ is the complex dielectric susceptibility of the material. The fields inside the sample are of constant magnitude and direction, but are phase-shifted in time relative to the external field. The **sample** shape factor, F_{sh}, is a real function of only the ratios of the lengths of the ellipsoid axis[4]. For a sphere, $F_{sh} = (1/3)$.

We have made the assumption that the functional form of Eq. (3) is still a good approximation, even for cylindrical samples, when considering the rms average field over the sample volume. For a particular cylindrical shape, the appropriate value of F_{sh} can be determined by measurement or by computation with SEAFISH. The agreement between measurement and SEAFISH computation for cylindrical samples has been confirmed[2] as long as the numerical approximations associated with the mesh size are valid.

The average energy absorbed per unit volume by a material is given by the usual formula

$$\frac{dP_{Av}}{dv_s} = \frac{1}{2}\omega\varepsilon_o\varepsilon_r'' |E_{\text{int}}|^2 , \tag{4}$$

where E_{int} is the field **inside** the sample, and the relative complex dielectric constant, ε_r and the complex dielectric susceptibility, χ, are related by

$$\varepsilon_r - 1 = \varepsilon_r' - 1 - j\varepsilon_r'' = \chi = \chi' - j\chi'' . \tag{5}$$

The average power absorbed by the sample for an approximately-uniform internal field is

$$P_{Av} \approx \frac{V_s\omega\varepsilon_o\chi''}{2} |E_{\text{int}}|^2 , \tag{6}$$

where E_{int} is the spacial rms average, peak-temporal field over the sample. Relating this to the field outside the sample by using Eq. (3), the power absorbed is

$$P_{Av} \approx \frac{V_s \omega \varepsilon_o \chi''}{2} * \frac{|E_{ext}(r \approx 0)|^2}{|1 + F_{sh}\chi|^2}, \tag{7}$$

where $E_{ext}(r \approx 0)$ is the external field in the region of the sample.

The contribution of the sample to the Q of the cavity may then be calculated using Eq. (1), (2) and (7), to give

$$Q \approx \frac{A}{4} \cdot \frac{V_c}{V_s} \cdot \frac{1}{\chi''} \cdot |1 + F_{sh}\chi|^2 = \frac{A}{4} \cdot \frac{V_c}{V_s} \cdot \frac{[(1 + F_{sh}\chi')^2 + (F_{sh}\chi'')^2]}{\chi''}. \tag{8}$$

Note that a large value of χ'', such that $\chi'' \gg \chi'$ (or tan $\delta \gg 1$), does not predict maximum absorption (*i.e.*, a minimum Q).

Equations (7) and (8) are significant refinements of the standard formula that are quoted for power absorption or sample contribution to Q. The inclusion of the sample shape factor dependence makes these useful for making engineering estimates of actual applicator designs. In the next section, the expression for the sample Q (Eq. (8)) is compared in detail with the exact computer calculations, and shape-factor values are determined which allow one to apply Eq. (3) to determine the mean, internal, sample field for a wide range of cylindrical sample shapes.

A COMPARISON OF THE SIMPLE ANALYTIC FORMALISM WITH EXACT CALCULATIONS

The contribution of the sample to the Q of the specified cavity (Fig. 1) was calculated for each of the three samples for a wide range of ε_r' and ε_r'' values. As an example, Fig. 6 shows

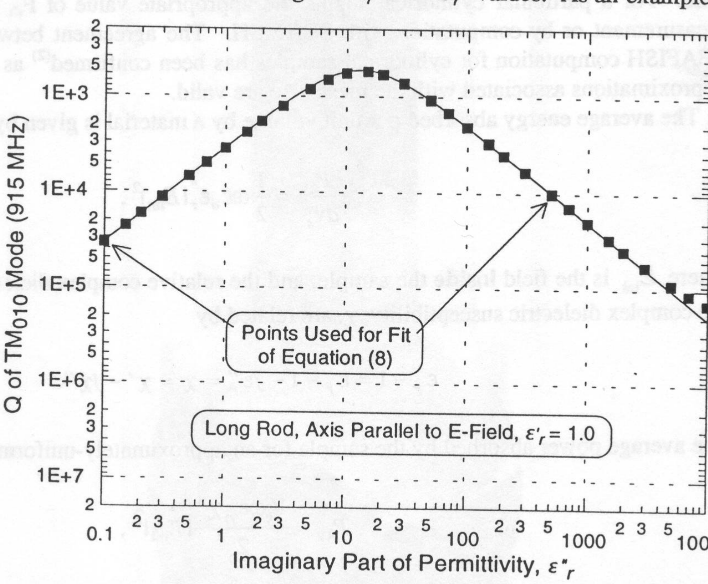


Figure 6. SEAFISH-calculated Q for the disc absorber (squares). The solid line is from Eq. (8), with A and F_{sh} determined by fitting to the calculations at the indicated points.

the exact solutions for Q for the disc absorber with $\varepsilon_r' = 1$, plotted against ε_r'', the relative dielectric loss factor. The analytic expression for Q (Eq. (8)) was fitted at the two specific points shown (Fig. 6), and the cavity electric field "A" values and sample shape factors were determined for the present re-entrant azimuthally symmetric cavity (Fig. 1). As the values of ε_r' and ε_r'' decrease, the influence of the sample shape factor in Eq. (8) decreases, so that the three different shaped samples have the same asymptotic dependence of Q on ε_r'' for $\varepsilon_r' = 1$. Hence, the value of Q at $\varepsilon_r'' = 0.1$ and $\varepsilon_r' = 1$ was used to determine the value of A for all samples at each frequency.

In the simple analytic theory, the value of the cavity electric field shape factor, A, is independent of sample shape. This is confirmed to good accuracy by the fit to the exact calculations. The present re-entrant cavity is not a simple cylinder in shape, and thus the A values are not those quoted for the theoretical cylinder cavity. The shape factor, F_{sh}, was determined by the fit at large ε_r'', avoiding however the very high ε_r'' region where the skin depth becomes significant. Obviously, the functional dependence of the analytic expression (Eq. (8)) reproduces the computer "data" very well except where skin depth or attenuation-length effects become important.

If one knows the value of the sample shape factor, F_{sh}, for a specific (l/d) ratio, then one can calculate the field inside a sample and thus the frequency and Q shift generated by the sample in a cavity. A series of SEAFISH runs was done for cylinders with a range of (l/d) ratios and the shape factors were determined (Fig. 7) and a fit interpolated. These shape factors are valid for simple cylinders with axis oriented along the electric-field direction in a field that is uniform over the sample dimensions. The degree of external field uniformity (without the sample) will determine the degree of approximation. For reference, the shape factors for ellipsoids of rotation are shown on Fig. 7, plotted against the ratio of the ellipsoid axes, (a/b). The difference between the shape factors is greatest just in the range of most practical samples.

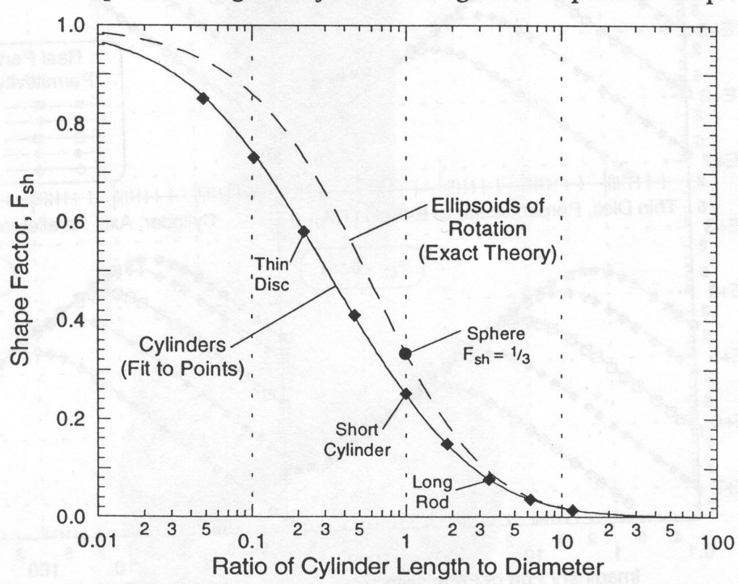


Figure 7. The sample shape factor, F_{sh}, for cylinders, determined by fitting Eq. 8 to SEAFISH calculations. The theoretical curve for ellipsoids of rotation is also shown (dotted line).

TABLE 1. The sample shape factors, F_{sh} and electric-field shape factors, A, for three cylindrical samples of the same volume but different length to diameter ratios in the re-entrant cavity (Fig. 1).

		Long Rod	Right Cylinder	Thin Disc
	(l/d)	3.43	1.000	0.219
915 MHz	A	0.703	0.697	0.696
TM_{010}-like	F_{sh}	0.0765	0.251	0.581
2450 MHz	A	0.477	0.473	0.474
TM_{020}-like	F_{sh}	0.0712	0.245	0.574

The SEAFISH-generated "data" for the three samples for a wide range of values of ε_r' and for both frequencies are shown in Fig. 8. For each frequency, the analytic curves were fitted using only the values of A and F_{sh} given in Table 1. The small systematic change of F_{sh} with

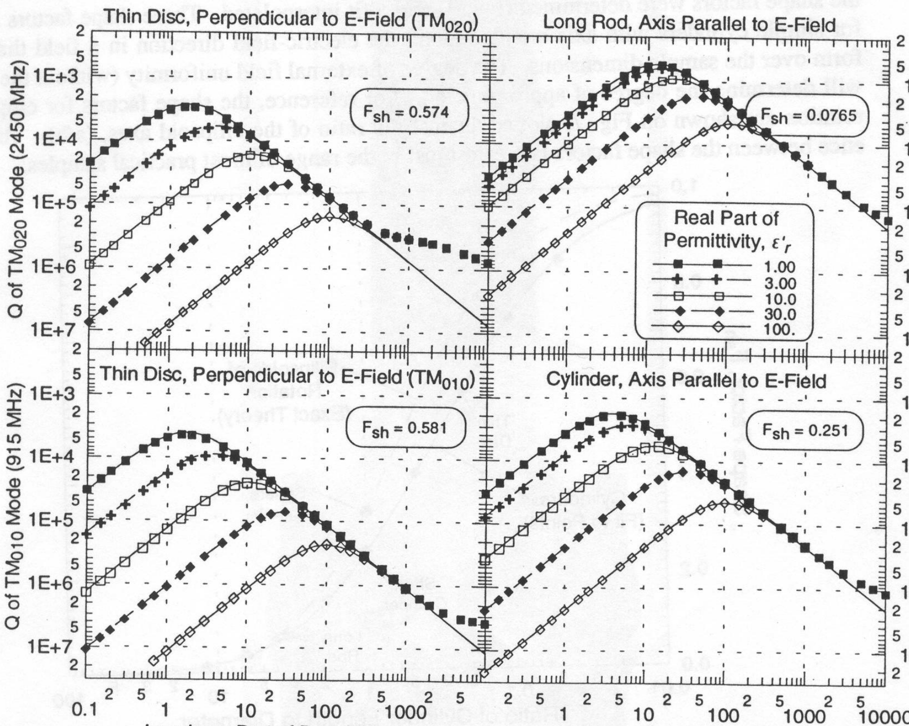


Figure 8. SEAFISH-calculated Q for the samples in the lossless cavity (Fig. 1). Note the inversion of the ordinate scale, to show maximum absorption in the sample as a peak.

increasing frequency was observed previously during experimental calibrations of a multi-mode, TM_{ono}, cylindrical cavity used for dielectric-perturbation measurements[5].

DISCUSSION

The calculations and analytic theory presented here apply accurately only to the situation of **small** samples of simple cylindrical shape in a relatively uniform, external electric field. In this case, the question of what is the best absorber material depends on the sample shape: a long rod aligned along the field absorbs best for $\tan\delta \geq 3$ and $\varepsilon' \leq 5$, while a flat disc aligned similarly has maximum absorption for $\tan\delta \approx 2$ and $\varepsilon' \approx 1$. As well, even the best "flat disc" absorber has almost an order of magnitude lower absorption per unit volume than the best long rod.

The practical application of microwave heating in industrial processes usually has constraints that exclude using the above-mentioned maximum absorption situation. Most materials have $\tan\delta$ much less than unity at room temperature, and only at elevated temperatures ($\approx$600°C for glasses, $\approx$1200°C for ceramics) does $\tan\delta$ exceed unity. Also, with the exception of some polymers and low atomic number ceramics, most materials have an intrinsic value of ε' greater than 5. Thus, achieving maximum absorption over a broad temperature range is rarely possible.

If one looks for the maximum absorption for an increasing value of ε', then the optimum value of $\tan\delta$ decreases to unity for both the disc and rod sample shapes. Thus, for example, a water solution ($\varepsilon' > 80$) should achieve its maximum power absorption per unit volume when its ionic impurity content is increased such that $\tan\delta \approx 1$ at the desired frequency. In this case, a small rod-shaped sample will absorb $\approx$50 times more power per unit volume than a thin disc sample. In a multimode-cavity heating situation the oven dimensions are at least a few wavelengths, and thus strong, internal electric-field variations occur for each mode, although the usual existence of many modes produces some field averaging. In this case, for small samples, the long rod shape will still absorb better, but with a reduced advantage due to the effective averaging over field orientations.

In summary, we have demonstrated that with a simple refinement of the standard formula for microwave power absorption one can obtain reasonable quantitative estimates of absorbed power and Q for various sample shapes and dielectric properties, and, as well, gain some understanding of the process.

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